Deriving KE Formula

Warning: may contain calculus!

$W = Fs$ (s = displacement, as opposed to d for distance...)

for small change δ; $F \to$ const as $\delta \to 0$

$\delta W = F\,\delta s$. Total W over all intervals:

$W = \int F\,ds$. Assume this causes acceleration to $\underline{v}$.

so $W = \int_0^v M \cdot \frac{dv}{dt}\,ds$ $\quad F = Ma = M \cdot \frac{dv}{dt}$

acc'n = 1st differential of vel.

but $v = \frac{ds}{dt}$

so

$W = \int_0^v M \frac{ds}{dt} \cdot dv = \int_0^v Mv\,dv$ can now be integrated!

$W = \frac{1}{2} M \left[v^2\right]_0^v = \frac{1}{2} M (v^2 - 0)$

$= \frac{1}{2} Mv^2$

Total $W = E_K$ so

$E_K = \frac{1}{2} Mv^2$